NDSU Math 166 Fall 2008 **Name**______________________________

Test 1 (Martin): Thursday September 25, 6-7:30 pm **Instructor**______________________________

You may use a graphing calculator on any problem.
Read each question carefully before you do any work. Be sure to answer what the question asks.
Use complete sentences to explain things. You must show valid work or reasoning to receive full credit for answers. All work and answers should be written in a large, blue examination booklet. Each of the problems is worth 10 points.

1. "True" and "False" questions. Support your responses:

(a) If f' is continuous on [1,3], then $\int_1^3 f'(v)dv = f(3) - f(1)$.

(b) The area of the circular region bounded by the equation $x^2 + y^2 = 1$ cannot be calculated using integrals since $x^2 + y^2 = 1$ is not a function.

(c) A definite integral of a positive function could always be interpreted as the area of a region and as the volume of a solid.

(d) $\int \sqrt{x^2+1}\, dx$ cannot be evaluated.

(e) $\dfrac{x^2+4}{x^2(x-4)}$ can be put in the form $\dfrac{A}{x^2} + \dfrac{B}{x-4}$.

2. Using right endpoints, write the n-term Riemann sum that corresponds to the integral $\int_1^4 x \ln x \, dx$.

3. Evaluate ANY TWO of these integrals. Be sure to identify the method you will use and show all steps. NO CREDIT WILL BE GIVEN FOR FINAL ANSWERS ALONE WITHOUT SUPPORTING WORK.

(a) $\int_0^1 x^2 e^x \, dx$ (b) $\int \sin^3 x \cos^2 x \, dx$ (c) $\int \dfrac{du}{u^2\sqrt{25-u^2}}$

4. Consider the integral $\int_0^1 \pi\left[\left(2-x^2\right)^2 - \left(2-x\right)^2\right] dx$. This integral could be interpreted as the volume of a solid of revolution, and also as the area enclosed by two curves.

(a) Describe a solid of revolution whose volume is given by this integral. You should use a carefully drawn and labeled sketch to help with your answer.

(b) Describe a region enclosed by two curves whose area is given by this integral. Again, use a carefully drawn and labeled sketch to help with your answer.

Problems 5-8 are printed on the back of this page

5. For ANY TWO of the following integrals, suggest a method of integration to evaluate and explain how and why it would work. DO NOT EVALUATE THE INTEGRALS.

(a) $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$ (b) $\int \frac{\sin^3 x}{\cos x}\,dx$ (c) $\int x^3 (\ln x)^2\,dx$

6. A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

7. Find the average value of the sine function on the interval $[0,\pi]$.

8. Find the total area enclosed by the curves $y = x - 3,\quad x = y^2 + 1$.

Take home problems (due at the start of recitation Tuesday September 30):

9. A parabolic dish is formed by rotating a parabola about a vertical axis. The dish is four feet deep along the vertical axis and the top radius is two feet. If the dish contains water to a depth of two feet, and water weighs 62.4 lb/ft, what amount of work is required to empty the water from the dish over the top?

10. (a) The base of a solid is a square with vertices located at (1,0), (0,1), (−1,0), (0,−1). Each cross-section perpendicular to the x-axis is a semicircle. Find the volume of the solid.
(b) Show that by cutting the solid of part (a), we can rearrange it to form a cone. Then show how to compute its volume more simply.

The take home problems are to be completed by your group. You should hand in one group solution with all group member's signatures, as with the special group homework problems. ***Late problems will not be accepted for grading.***

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